Tutorial Assignment 4: Superfluids and Superconductors

Handwriting should be legible. Calculations should be explained. Units should be given. Numerical answers should be given to 4 significant figures.

1. (i) Using chemical potential $\mu = -1$ J. At temperature where $k_BT = 2$ J, calculate the occupancy $f(\varepsilon)$ of bosons at energies $\varepsilon = 0, 0.5, 1, 1.5, 2$ J. Plot a graph of $f(\varepsilon)$ against ε. [1]

(ii) Repeat this with $k_BT = 1$ J on the same graph. Explain what this means about the number of bosons if μ remains constant as T falls. [1]

(iii) Plot on a separate piece of paper the graph for $k_BT = 1$ J and $\mu = -0.4$ J. Copy the graph from (ii) to this graph. Explain what this means about the number of bosons if μ is increased at constant T.[2]

(iv) On a separate piece of paper, copy graph (i) and graph (iii) to the same graph. What does this suggest about μ if the bosons are to remain constant as T falls? [2]

(v) There is one mole of helium-4 atoms, with a volume of 27.6 cm^3. Assume that these are bosons. Calculate the number of excited atoms at T = 0, 2, 4, 6 K. Plot a graph of this number in moles. [2]

(vi) On the same graph, plot the graph for the constant number of 1 mole. Label the parts of the graphs that are not valid and explain why. Find the condensation temperature from the graph. [2]

2. (i) A 10 cm long niobium cylinder, with a radius of 1 cm, sits in a 0.01 T magnetic field that is parallel to its axis. Given that the cylinder is superconducting, and that the field penetrates to a depth of 300 Å, estimate the flux in the cylinder. Explain with a diagram. [2]

(ii) Using the relation between magnetic field and current in a solenoid, find the total current circulating in the surface. [2]

3. (i) An electron passing close to a positive ion in niobium attracts and displaces it. Estimate the natural frequency this ion. (Sound speed is 3480 m/s. Molar volume is 10.84 cm^3.) [1]

(ii) How long does it take for the ion to return to its rest position? [1]

(iii) If the electron is at the Fermi energy E_F, find its velocity (assuming one free electron per atom). [2]

(iv) This electron leaves behind a trail of displaced ions. Estimate the length of this trail in terms of the spacing between atoms of 3.3 Å. What does this suggest about the Cooper pair? [2]

CONSTANTS

Speed of light in vacuum		c	=	3.00×10^{8} ms^{-1}
Permeability of vacuum		μ_0	=	$4\pi \times 10^{-7}$ Hm^{-1}
			=	$4\pi \times 10^{-7}$ VsA^{-1}m^{-1}
Permittivity of vacuum		ε_0	=	8.85×10^{-12} Fm^{-1}
			=	8.85×10^{-12} AsV^{-1}m^{-1}
Elementary charge		e	=	1.60×10^{-19} C
Planck constant		h	=	6.63×10^{-34} Js
	$h/2\pi =$	$\hbar$	=	1.05×10^{-34} Js
Avogadro constant		N_A	=	6.02×10^{23} mol^{-1}
Boltzmann constant		k_B	=	1.38×10^{-23} JK^{-1}
Gas constant		R	=	8.31 JK^{-1}mol^{-1}
Unified atomic mass constant		m_u	=	1.66×10^{-27} kg
			=	931.5 MeVc^{-2}
Electron mass		m_e	=	9.11×10^{-31} kg
Proton mass		m_p	=	1.67×10^{-27} kg
Gravitational constant		G	=	6.67×10^{-11} Nm^{2}kg^{-2}
Acceleration due to gravity		g	=	9.8 ms^{-2}
Bohr magneton		μ_B	=	9.27×10^{-24} JT^{-1}